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One-loop Effective Actions in Shape-invariant Scalar Backgrounds

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ABSTRACT

The field-theoretic one-loop effective action in a static scalar background depending nontrivially on a single spatial coordinate is related, in the proper-time formalism, to the trace of the evolution kernel (or heat kernel) for an appropriate, one dimensional, quantum-mechanical Hamiltonian. We describe a recursive procedure applicable to these traces for shape-invariant Hamiltonians, resolving subtleties from the continuum mode contributions by utilizing the expression for the regularized Witten index. For some cases which include those of domain-wall-type scalar backgrounds, our recursive procedure yields the full expression for the scalar or fermion one-loop effective action in both (1+1) and (3+1)-dimensions.

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The calculation of the one-loop effective action or one-loop quantum correction to the vacuum energy in a static background configuration is an important problem in renormalizable field theory which received much attention over the years [1]-[5]. If the background classical configuration has a nontrivial spatial dependence as in the case of a soliton background, it becomes a rather difficult task. This is because one must evaluate certain nontrivial functional determinant to determine the effect due to fluctuations of quantum fields in the presence of such a background configuration. Renormalization aspect is now well understood, however.

In this paper we present some interesting observations which are relevant for the scalar or fermion one-loop effective action in a static scalar background having non-trivial dependence on a single spatial coordinate only. Specifically, we shall employ the Schwinger proper-time representation [1]. Then the principal task in determining the one-loop effective action reduces to the calculation of the trace of the evolution kernel (or heat kernel) for a suitable, one-dimensional, quantum-mechanical Hamiltonian. We are here interested in a family of background scalar fields which lead, in this proper-time formalism, to so-called shape-invariant Hamiltonians [6] (as formulated on the basis of supersymmetric quantum mechanics [7]). For a family of shape invariant Hamiltonians, we will below derive the recursive relation applicable to the corresponding traces; this may in turn be used to relate the one-loop effective actions for different scalar backgrounds in the given family. As will be considered in the latter portion of this paper, applying this scheme to the one-loop energies of kink configurations produces certain results of direct physical interest.

First let us consider a system involving a quantized Dirac field in the presence of an external scalar background $\tilde{\phi}(x)$, described by the Lagrangian

$$\mathcal{L} = \bar{\psi}(x)(i\gamma^\mu\partial_\mu + \phi(x))\psi(x), \quad (\phi \equiv -m + \tilde{\phi}(x)). \tag{1}$$

The (unrenormalized) effective action $W_0(\phi)$ may then defined through the path integral

$$e^{iW_0(\phi)} = N\int [D\psi][D\bar{\psi}]\exp\left(i\int d^dx\mathcal{L}\right), \tag{2}$$

where d is the space-time dimension and N the normalization factor chosen such that $W_0(\phi)$ may vanish if there is no background field for instance. In the Schwinger proper-time representation [1], it can be expressed by the form

$$W_0(\phi) = \frac{i}{2}\int_0^\infty \frac{ds}{s}e^{-s\epsilon}\int d^dx\,\mathrm{tr}\,\langle xs|x\rangle, \tag{3}$$

up to a suitable, normalization-related, additive constant. Here, $\langle xs|x\rangle$ is the coincidence limit of the proper-time Green's function,

$$\langle xs|x'\rangle = \langle x|e^{-is\mathcal{H}}|x'\rangle, \tag{4}$$

for the *squared* Dirac operator $\mathcal{H}$ which has the explicit form

$$\mathcal{H} = -\partial^2 + \phi^2 + i\gamma^\mu \partial_\mu \phi. \tag{5}$$

We are here interested in the case when the background field $\tilde{\phi}$ depends only on a single spatial coordinate which is denoted as z. Then the above 'Hamiltonian' $\mathcal{H}$ has an interesting structure: aside from the trivial transverse derivative terms (including the time derivative), $\mathcal{H}$ defines a one-dimensional supersymmetric quantum mechanics [8] with superpotential ϕ. In the basis where $\gamma^\mu \partial_\mu z$ is given by a diagonal matrix, $\mathcal{H}$ becomes

$$\begin{aligned} \mathcal{H} &= -\partial_\perp^2 + \begin{pmatrix} -\partial_z^2 + \phi^2 + \phi' & 0 \\ 0 & -\partial_z^2 + \phi^2 - \phi' \end{pmatrix} \\ &= -\partial_\perp^2 + \begin{pmatrix} \mathcal{H}_+ & 0 \\ 0 & \mathcal{H}_- \end{pmatrix} \end{aligned} \tag{6}$$

where $\phi' \equiv d\phi/dz$ and $\partial_\perp^2 \equiv \partial^2 - \partial_z^2$. The one-dimensional Hamiltonians $\mathcal{H}_\pm$ form a supersymmetric pair and can be written as

$$\mathcal{H}_+ = QQ^\dagger, \quad \mathcal{H}_- = Q^\dagger Q, \tag{7}$$

where

$$Q = -i\frac{d}{dz} - i\phi, \quad Q^\dagger = -i\frac{d}{dz} + i\phi. \tag{8}$$

We also remark that, in a *static* background field, the effective action per unit time interval coincides, up to the sign, with the one-loop energy of the system defined by the background configuration [9].

In order to obtain the effective action W_0 in (3), we need to evaluate the quantity

$$\mathrm{Tr}\, e^{-is\mathcal{H}_+} + \mathrm{Tr}\, e^{-is\mathcal{H}_-}, \tag{9}$$

where we defined Tr $\equiv \int dz\, \mathrm{tr}$. An important property of the supersymmetric system is that the two Hamiltonians $\mathcal{H}_\pm$ are almost isospectral, that is, they share all the eigenvalues except for the zero eigenvalue of, say, $\mathcal{H}_-$. This enables us to relate one of the above traces with the other through the celebrated (regularized) Witten index $\Delta(is)$ [10],

$$\Delta(is) = \mathrm{Tr}\, e^{-is\mathcal{H}_-} - \mathrm{Tr}\, e^{-is\mathcal{H}_+}. \tag{10}$$

Essentially $\Delta(is)$ counts the difference between the number of zero modes of $\mathcal{H}_-$ and $\mathcal{H}_+$. There are, however, some subtleties from the continuum mode contributions, which can make the index fractional [11]. Nevertheless, it is known that the regularized Witten index is a topological invariant, i.e., is invariant under any compact perturbations of the background field. (For a particularly simple proof of this fact, see [12].) Hence it can be calculated largely independently of the details of the Hamiltonian. Indeed, for any background field $\phi(z)$ which smoothly interpolate between two nonzero values $\phi_\pm$ at $z = \pm\infty$, the Witten index is given purely in terms of $\phi_\pm$ [11],

$$\Delta(is) = \frac{\mathrm{sgn}\,(\phi_+)}{2}\left[1 - \frac{1}{\sqrt{\pi}}\Gamma(1/2, is\phi_+^2)\right] - \frac{\mathrm{sgn}\,(\phi_-)}{2}\left[1 - \frac{1}{\sqrt{\pi}}\Gamma(1/2, is\phi_-^2)\right], \tag{11}$$

where $\Gamma(a,x)$ is the incomplete gamma function. Therefore we need only to evaluate one of the traces, say, $\mathrm{Tr}\, e^{-is\mathcal{H}_+}$.

Similar consideration can also be given to the effective action for a quantized scalar field which is described by the Lagrangian

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - U(\varphi). \tag{12}$$

The one-loop effective action due to quantum fluctuations of the scalar field in the presence of the static background field $\phi(z)$ becomes

$$W_0(\phi) = -\frac{i}{2}\int_0^\infty \frac{ds}{s} e^{-s\epsilon}\int d^dx \langle x|e^{-is\mathcal{H}}|x\rangle, \tag{13}$$

where

$$\begin{aligned}\mathcal{H} &= -\partial_\perp^2 - \partial_z^2 + U''(\phi(z)) \\ &\equiv -\partial_\perp^2 + \mathcal{H}_s.\end{aligned} \tag{14}$$

Clearly, we are led to consider a quantum mechanical Hamiltonian analogous to that of the Dirac field case.

So far we have seen that, in determining the one-loop effective action in a static scalar background, it becomes necessary to calculate the trace of the evolution kernel for a certain quantum mechanical Hamiltonian. In general, such a task cannot be performed exactly and one has to rely on certain approximation methods such as the WKB method. For a certain class of scalar background $\phi(z)$, however, the resulting quantum mechanical system is exactly solvable and it has been known that such a system usually possesses a special 'symmetry' known as shape invariance [6]. We shall call such a scalar background simply as a shape-invariant scalar background. In what follows, our direct concern will be on the trace of the evolution kernel in such shape-invariant scalar background.

Let c_0 be a parameter appearing in the background scalar field $\phi(z)$, and then we have the Hamiltonians $\mathcal{H}_\pm(c_0)$ defined by (7) and (8), Here, $\mathcal{H}_+(c_0)$ is called shape invariant [6] when

$$\begin{aligned} \mathcal{H}_+(c_0) &= Q(c_0)Q^\dagger(c_0) \\ &= Q^\dagger(c_1)Q(c_1) + R_1 = \mathcal{H}_-(c_1) + R_1, \end{aligned} \tag{15}$$

where c_1 is a new parameter depending on c_0 and R_1 is another constant. Now we can construct a sequence of Hamiltonians $\mathcal{H}^{(n)}$ such that

$$\begin{aligned} \mathcal{H}^{(0)} &= \mathcal{H}_-(c_0) \\ \mathcal{H}^{(1)} &= \mathcal{H}_+(c_0) = \mathcal{H}_-(c_1) + R_1 \\ &\cdots \\ \mathcal{H}^{(n)} &= \mathcal{H}_+(c_{n-1}) + \sum_{k=1}^{n-1} R_k \\ &= \mathcal{H}_-(c_n) + \sum_{k=1}^{n} R_k. \end{aligned} \tag{16}$$

Note that each pair $\mathcal{H}_+(c_k)$ and $\mathcal{H}_-(c_k)$ itself defines a new supersymmetric quantum mechanical system, and the corresponding Witten index

$$\Delta_k(is) = \mathrm{Tr}\, e^{-is\mathcal{H}_-(c_k)} - \mathrm{Tr}\, e^{-is\mathcal{H}_+(c_k)} \tag{17}$$

may be calculated separately without much effort using (11). Then (16) allows us to relate the trace of the evolution kernel for $\mathcal{H}_+(c_0)$ to that for $\mathcal{H}_+(c_n)$ as

$$\begin{aligned} \mathrm{Tr}\, e^{-is\mathcal{H}_+(c_0)} &= e^{-isR_1}\mathrm{Tr}\, e^{-is\mathcal{H}_-(c_1)} \\ &= e^{-isR_1}\left[\mathrm{Tr}\, e^{-is\mathcal{H}_+(c_1)} + \Delta_1(is)\right] \\ &= \cdots \\ &= e^{-isE_n}\mathrm{Tr}\, e^{-is\mathcal{H}_+(c_n)} + \sum_{k=1}^{n} e^{-isE_k}\Delta_k(is), \end{aligned} \tag{18}$$

where $E_k = \sum_{i=1}^{k} R_i$. Therefore, once the trace is calculated for one shape-invariant Hamiltonian, the traces for the rest of Hamiltonians in the same shape-invariant family can be obtained immediately from (18) without new calculations. This is especially useful when for some value of n the Hamiltonian $\mathcal{H}_+(c_n)$ reduces to a simple one. It will then allow us to calculate the effective action (3) almost effortlessly for certain nontrivial scalar backgrounds.

Similar development can be given for the trace defined by the scalar Hamiltonian $\mathcal{H}_s$, if the potential term $U''(\phi)$ produces a shape-invariant potential. Of course, the background field ϕ is no longer identified as the superpotential appearing in the operator Q; but, with a suitable superpotential, say, $\bar{\phi}$, the Hamiltonian may still be written in a shape-invariant form. This is the case when the background field ϕ corresponds to the kink solution of the scalar theory.

With the trace of the evolution kernel at hand, one still has to perform the integration over the proper time s to obtain the effective action via (3) or (13). In doing so, ultraviolet divergences appear from the $s \to 0$ end — the expression should be renormalized. Here one may utilize the Dewitt WKB expansion [13]

$$s \to 0 : \langle x | e^{-is\mathcal{H}} | y \rangle = \frac{i}{(4\pi i s)^{d/2}} e^{i(x-y)^2/4s - im^2 s} \sum_{k=0}^{\infty} a_k(x,y)(is)^k, \tag{19}$$

when m is the mass parameter of the corresponding free theory. The coefficient functions $a_k(x,y)$ are well-behaved in the region including the coincidence point $x = y$ and can be determined using the Schrödinger-type equation satisfied by the evolution kernel. For

$$\mathcal{H} = -\partial^2 + m^2 + V, \tag{20}$$

the coincidence limits of first few coefficient functions are given by [14]

$$a_0(x,x) = 1, \quad a_1(x,x) = -V, \quad a_2(x,x) = -\frac{1}{6}\partial^2 V + \frac{1}{2}V^2. \tag{21}$$

It is now clear from (3) or (13) that all the divergences result from the terms involving the coincidence limits $a_0(x,x), \ldots, a_{[d/2]}(x,x)$ in the series for $\langle xs|x \rangle$, and the renormalized effective action can be obtained if those terms are subtracted away[1].

As an illustration, we calculate the fermion effective action in (1+1) dimensions when the background field $\phi(z)$ is given by the form

$$\phi(z) = c_0 \tanh z. \tag{22}$$

[1]Here the dimensional regularization is implicitly assumed. In general, the precise forms of the subtraction terms will depend on the renormalization scheme used.

This is a typical situation for a fermion having the Yukawa-type interaction with the scalar field corresponding to the kink configuration. (See (39) below.) It is easy to see that it generates a family of shape-invariant Hamiltonians $\mathcal{H}_{\pm}^{(k)}$ with superpotentials

$$\phi^{(k)} = c_k \tanh z, \quad c_k = c_0 - k. \tag{23}$$

The constant R_k defined in (16) is given by

$$R_k = c_{k-1}^2 - c_k^2 = 2(c_0 - k) + 1. \tag{24}$$

Now, for systems defined by the superpotentials in (23), we may use the formula (18) in conjunction with (10) to get the relationship between the traces of the corresponding evolution kernels. In particular, if the constant c_0 appearing in $\phi(z)$ is a positive integer, i.e.,

$$c_0 = n, \tag{25}$$

the n-th superpotential $\phi^{(n)}$ vanishes and the corresponding Hamiltonian becomes a *free* Hamiltonian. In such situation, one can readily obtain the effective action in the given nontrivial background by using our relationship. Explicitly, from (3), the unrenormalized effective action is then given by

$$\begin{aligned} W_0 &= \frac{i}{2^{2-d/2}} \int_0^\infty \frac{ds}{s} \int d^{d-1}x_\perp \langle x_\perp | e^{is\partial_\perp^2} | x_\perp \rangle \,\mathrm{Tr}\,(e^{-is\mathcal{H}_-} + e^{-is\mathcal{H}_+}) \\ &= \frac{i\mathcal{V}_\perp}{2^{2-d/2}} \frac{i}{(4\pi i)^{(d-1)/2}} \int_0^\infty \frac{ds}{s^{(d+1)/2}} \left[2e^{-isE_n}\,\mathrm{Tr}\, e^{is\partial_z^2} + 2\sum_{k=1}^{n} e^{-isE_k} \Delta_k(is) + \Delta_0(is) \right], \end{aligned} \tag{26}$$

where $\mathcal{V}_\perp$ is the space-time volume of the transverse directions and E_k is given by

$$E_k = 2nk - k^2. \tag{27}$$

The Witten index $\Delta_k(is)$ is calculated using the formula (11) as

$$\Delta_k(is) = 1 - \frac{1}{\sqrt{\pi}} \Gamma(1/2, is(n-k)^2). \tag{28}$$

The first term in the integrand of (26) produces the effective action appropriate to the free theory with mass $m = n(= c_0)$. It may be cancelled by the normalization factor N in (2) and we subtract it from the effective action. Even after such subtraction, the remaining expression is still divergent; to secure a finite expression, further renormalization must be considered.

Let us first consider the (1+1) dimensional case. We here use the procedure described already, i.e., apply the Dewitt WKB expansion (19) to the squared Dirac operator with $\phi(z)$ in (22), writing

$$-\partial_\perp^2 + \mathcal{H}_\pm = -\partial^2 + m^2 + V_\pm, \tag{29}$$

where $m^2 = n^2 (= c_0^2)$ is the free particle mass and

$$V_\pm = \phi^2 \pm \phi' - m^2. \tag{30}$$

Now the divergent terms are cancelled by the subtraction of the $a_1^\pm(x,x)$ term in (21), that is, by considering the counterterm

$$\delta W_c = \frac{i}{2}\int_0^\infty \frac{ds}{s} e^{-s\epsilon} \int d^dx \langle x| e^{-is(-\partial^2+m^2)}|x\rangle is[a_1^+(x,x) + a_1^-(x,x)], \tag{31}$$

with

$$\begin{aligned} a_1^+(x,x) + a_1^-(x,x) &= -(2\phi^2 - 2m^2) \\ &= 2n^2 \operatorname{sech}^2 z. \end{aligned} \tag{32}$$

Then the renormalized effective action is given by

$$\begin{aligned} W &= -\frac{T}{2(4\pi i)^{1/2}} \int_0^\infty \frac{ds}{s^{3/2}} \left\{ 2\sum_{k=1}^{n} e^{-is(2nk-k^2)} \left[1 - \frac{1}{\sqrt{\pi}}\Gamma(\frac{1}{2}, is(n-k)^2)\right] \right. \\ &\qquad \left. - \frac{1}{\sqrt{\pi}}\Gamma(\frac{1}{2}, isn^2) - \frac{e^{-isn^2}}{\sqrt{4\pi is}} \int dz\, 2isn^2 \operatorname{sech}^2 z \right\}, \end{aligned} \tag{33}$$

where $T = \mathcal{V}_\perp \equiv \int dt$ and we have ignored the zero mode contribution in $\Delta_0(is)$ as usual [9]. The integral over s can be performed explicitly and we obtain

$$W = -T\left[\frac{2}{\pi}\sum_{k=1}^{n-1}\sqrt{n^2-k^2}\tan^{-1}\left(\frac{\sqrt{n^2-k^2}}{k}\right) - \sum_{k=1}^{n-1}\sqrt{n^2-k^2} + \frac{n^2}{\pi}\right]. \tag{34}$$

The result is in exact agreement with the one-loop energy correction calculated in [15].

In (3+1) dimensions, on the other hand, we need to subtract terms involving the coefficients $a_2(x,x)$ also. The counterterm is thus given by

$$\begin{aligned} \delta W_c &= \frac{i}{2}\int_0^\infty \frac{ds}{s} e^{-s\epsilon} \int d^dx \langle x| e^{-is(-\partial^2+m^2)}|x\rangle \\ &\qquad \times \left\{ is[a_1^+(x,x) + a_1^-(x,x)] + (is)^2[a_2^+(x,x) + a_2^-(x,x)] \right\}, \end{aligned} \tag{35}$$

and, for $\phi(z) = n \tanh z$, we have (from (21))

$$a_2^+(x,x) + a_2^-(x,x) = \frac{n^2}{3}(2\cosh 2z + 3n^2 - 1)\text{sech}^4 z. \tag{36}$$

Based on this information, we then obtain the renormalized effective action

$$\begin{aligned} W &= W_0 - \delta W_c \\ &= -\frac{\mathcal{V}_\perp}{3\pi^2}\left[-2\sum_{k=1}^{n-1}(n^2-k^2)^{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{n^2-k^2}}{k}\right) + \pi\sum_{k=1}^{n-1}(n^2-k^2)^{\frac{3}{2}} - \frac{1}{3}n^2(n^2-2)\right]. \end{aligned} \tag{37}$$

Next, we consider the spontaneously broken φ^4-type scalar field theory with the potential energy function

$$U(\varphi) = \frac{\lambda}{4}(\varphi^2 - m^2/\lambda)^2, \tag{38}$$

and calculate the one-loop effective action in the background of a kink solution

$$\phi(z) = \frac{m}{\sqrt{\lambda}}\tanh\frac{m}{\sqrt{2}}z. \tag{39}$$

Rescaling the coordinate $z \to \sqrt{2}z/m$, we find that the one-dimensional quantum mechanical Hamiltonian $\mathcal{H}_s$ in (14) is given by

$$\mathcal{H}_s = -\partial_z^2 + 6\tanh^2 z - 2. \tag{40}$$

This corresponds to the shape invariant Hamiltonian with the superpotential $\bar{\phi} = 2\tanh z$ (and so $\bar{\phi}^{(2)} = 0$. See (23). In other words, this is actually a special case of the previous Dirac system and we can immediately obtain the desired renormalized one-loop effective action in an entirely similar manner. The result in (1+1) dimensions for the scalar one-loop effective action reads

$$W = Tm\left(\frac{3}{\sqrt{2}\pi} - \frac{1}{2\sqrt{6}}\right), \tag{41}$$

which is in agreement with the result in [2]. In (3+1) dimensions our procedure produces the following result:

$$W = \mathcal{V}_\perp\frac{\sqrt{3}m^3}{24\sqrt{2}\pi}. \tag{42}$$

The same result was also obtained in a recent paper [16].

Finally, we comment on the (0+1) dimensional (quantum mechanical) case where the kink configuration corresponds to an instanton solution and the determinant factor is then related to the tunneling rate of the system [17]. The determinant has been calculated in [18] utilizing the shape invariance and heat kernel in a somewhat similar fashion to the present work. However, our method is much simpler since we have made more effective use of supersymmetry, especially, by utilizing the expression for the Witten index with which most of the calculations have been dispensed with.

In summary, we employed the Schwinger proper-time formalism to represent the scalar or fermion one-loop effective action in a static scalar background. It is related to the trace of the evolution kernel for an appropriate quantum mechanical Hamiltonian. In the case that the scalar background results in a shape-invariant system, we derived a recursive relation applicable to the traces for shape-invariant Hamitonians. Most of the complications in the calculation disappear if the expression for the regularized Witten index is used in a judicious way. We applied this scheme to the scalar or fermion system in a kink background, to obtain the corresponding one-loop effective actions in (1+1) and (3+1) dimensions explicitly.

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